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Retarded functions in noncommutative theories

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Abstract

The perturbative approach to quantum field theory using retarded functions is extended to noncommutative theories. Unitarity as well as quantized equations of motion are studied and seen to cause problems in the case of space–time noncommutativity. A modified theory is suggested that is unitary and preserves the classical equations of motion on the quantum level.

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1. Introduction

Noncommutative Quantum Field Theory (NCQFT) has recently received renewed attention (see [1] for a review). This interest is triggered by its appearance in the context of string theory [2], and by the observation that Heisenberg's uncertainty principle along with general relativity suggests the introduction of noncommutative space–time [3]. Its mathematical foundations may also be found in Connes' formulation of noncommutative geometry, Moyal noncommutative field theory has been shown to be compatible with the latter one in the Euclidean case [4]. Moreover, it arises in the framework of deformation quantization [5].

Coordinates are considered as noncommuting hermitian operators $\hat{x}^\mu$, which satisfy the commutation relation

$$\left[\hat{x}^\mu, \hat{x}^\nu\right] = i\theta^{\mu\nu}. \tag{1}$$

We will assume the antisymmetric matrix $\theta^{\mu\nu}$ to be constant. The algebra of these noncommuting coordinate operators can be realized on functions on the ordinary Minkowski space by introducing the Moyal $\star$-product

$$(f \star g)(x) = e^{\frac{i}{2}\theta^{\mu\nu}\partial_\mu^\xi\partial_\nu^\eta} f(x+\xi)g(x+\eta)\big|_{\xi=\eta=0}. \tag{2}$$

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To obtain a NCQFT from a commutative QFT, one replaces the ordinary product of field operators by the star product in the action. Due to the trace property of the star product, meaning that

$$\int dx\,(f_1 \star \cdots \star f_n)(x) \tag{3}$$

is invariant under cyclic permutations, the free theory is not affected and noncommutativity only appears in the interaction part. As an example, the interaction in noncommutative $\varphi^3_\star$-theory reads

$$S_{\text{int}} = \frac{g}{3!}\int dx\,(\varphi \star \varphi \star \varphi)(x). \tag{4}$$

A first suggestion for perturbation theory has been made in [6], where the Feynman rules for the ordinary QFT are only modified by the appearance of momentum-dependent phase factors at the vertices. These are of the form $e^{-ip\wedge q}$, with $p \wedge q = \frac{1}{2} p_\mu \theta^{\mu\nu} q_\nu$. In the case of only space–space noncommutativity, i.e., $\theta^{0i} = 0$, this approach leads to the UV/IR mixing problem, a renormalizable model has been suggested in [7]. The general case of space–time noncommutativity, i.e., $\theta^{0i} \neq 0$, raises problems at an earlier stage due to the nonlocality of the star product, which involves time-derivatives to arbitrary high orders. It has been shown that the S-matrix is no longer unitary as the cutting rules are violated [8], the corresponding calculation involves only the tree level and the finite part of the one-loop level.

To cure this problem, a different perturbative approach, TOPT, has been suggested for scalar theories in [9]. It mainly builds on the observation that for space–time noncommutativity time-ordering and star product of operators are not interchangeable, their order matters. Defining TOPT by carrying out time-ordering *after* taking star products, a manifestly unitary theory is obtained.

However, further problems arise. The explicit violation of causality inside the region of interaction was discussed in [10], however, this alone does not spoil the consistency of the formalism. In [11] it has been shown that Ward identities in NCQED are violated if TOPT is applied, which could be traced back to altered current conservation laws on the quantized level [12]. Moreover, remaining Lorentz symmetry, i.e., Lorentz transformations, which leave the noncommutativity parameter $\theta^{\mu\nu}$ invariant, is not respected by TOPT [13].

To formulate a consistent perturbative approach to space–time noncommutative theories is thus still a task to work on. One recent suggestion building on the observation of violated remaining Lorentz symmetry in TOPT has been made in [14], another one starts from the Yang–Feldman equations [15]. In this Letter we want to investigate the approach via retarded functions as introduced in the commutative case in [16] and further elaborated in [17], a pedagogical presentation may also be found in [18]. In this formalism, retarded functions are used instead of time-ordered Green's functions, the motivation is that the usage of the first ones allows an easier derivation of unitarity and causality due to certain support properties of retarded functions. We will extend this approach in a natural way to noncommutative theories and investigate unitarity as well as quantized equations of motion. The latter is motivated by its similarity to current conservation laws: if classical equations of motion are not altered on the quantum level also classical current conservation laws will remain valid on the quantized level. We will find both unitarity as quantized equations of motion to be disturbed in a specific way that allows to modify the theory such that it is unitary and preserves the classical equations of motion on the quantum level.

2. The commutative case

2.1. Retarded functions and the generating functional

We consider a field theory with a single hermitian field ϕ of mass m. The retarded products are then given by retarded multiple commutators of ϕ:

$$R(x;x_1,\ldots,x_n) = (-i)^n \sum_{\text{perm}} \vartheta\big(x^0 - x_1^0\big)\cdots\vartheta\big(x_{n-1}^0 - x_n^0\big)\big[\cdots\big[\phi(x),\phi(x_1)\big]\cdots\phi(x_n)\big], \tag{5}$$

where the summation is taken over all permutations of the n coordinates x_i, ϑ denotes the step function. The support property $R(x; x_1, \ldots, x_n) \neq 0$ only for $x^0 \geqslant x_1^0, \ldots, x_n^0$ is immediately clear from this definition. The retarded functions are now defined as the vacuum expectation values of the retarded products,

$$r(x; x_1, \ldots, x_n) = \langle 0|R(x; x_1, \ldots, x_n)|0\rangle, \tag{6}$$

and with their help the S-matrix may be obtained by a reduction formula as elaborated by Lehmann, Symanzik and Zimmermann in [16], the amputation of external legs works as usual through multiplication by the inverse propagator.

To compute retarded functions we follow [18] and introduce the generating functional

$$\begin{aligned}\mathcal{R}[j', j] = \exp&\left\{2\int dx\,\sin\left(\frac{1}{2}\frac{\delta}{\delta j(x)}\frac{\delta}{\delta\frac{\delta}{\delta j'(x)}}\right)\int dy\,\mathcal{L}_{\text{int}}\left(\frac{\delta}{\delta j'(y)}\right)\right\}\\ &\times\exp\left\{\int dz\,dw\left(\frac{1}{4}j'(z)\Delta^{(1)}(z-w)j'(w) - j'(z)\Delta^{\text{ret}}(z-w)j(w)\right)\right\},\end{aligned} \tag{7}$$

where Δ^{ret} is a Green's function to the Klein–Gordon equation

$$\Delta^{\text{ret}}(x) = \lim_{\epsilon\to+0}\frac{-1}{(2\pi)^4}\int d^4k\,\frac{e^{-ikx}}{(k+i\epsilon)^2 - m^2} \tag{8}$$

with the support property $\Delta^{\text{ret}}(x) = 0$ for $x^0 < 0$ and $\Delta^{(1)}$ is given by

$$\Delta^{(1)}(x) = \frac{1}{(2\pi)^3}\int d^4k\,\delta\big(k^2 + m^2\big)e^{-ikx} \tag{9}$$

being a solution to the homogeneous Klein–Gordon equation: $(\Box + m^2)\Delta^{(1)}(x) = 0$.

Retarded functions are obtained by means of functional differentiation:

$$r(x; x_1, \ldots, x_n) = \frac{\delta}{\delta j'(x)}\frac{\delta^n}{\delta j(x_1)\cdots\delta j(x_n)}\mathcal{R}[j', j]\bigg|_{j'=j=0}. \tag{10}$$

2.2. Diagrammatic rules

For later purpose, we want to write the outcome of Eq. (10) in the form of diagrams. Its lines will obviously carry Δ^{ret} or $\Delta^{(1)}$, and for $r(x; x_1, \ldots, x_n)$ there will be endpoints $x, x_1, \ldots, x_n$.

To see which diagrams are allowed according to (10), we expand the first exponential in (7) in the example of $\mathcal{L}_{\text{int}} = g\phi^m$:

$$\begin{aligned}\mathcal{R}[j', j] = 1 + \sum_{n=1}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}&\int dy_i\,2\sin\left(\frac{1}{2}\frac{\delta}{\delta j(y_i)}\frac{\delta}{\delta\frac{\delta}{\delta j'(y_i)}}\right)\int dz_i\,g^m\frac{\delta^m}{\delta j'(z_i)^m}\\ &\times\exp\left\{\int dy\,dz\left(\frac{1}{4}j'(y)\Delta^{(1)}(y-z)j'(z) - j'(y)\Delta^{\text{ret}}(y-z)j(z)\right)\right\}.\end{aligned} \tag{11}$$

Recalling that

$$r(x; x_1, \ldots, x_n) = \frac{\delta}{\delta j'(x)}\frac{\delta^n}{\delta j(x_1)\cdots\delta j(x_n)}\mathcal{R}[j', j]\bigg|_{j'=j=0} \tag{12}$$

we see that x is connected by $\Delta^{\text{ret}}(x-a)$ or $\Delta^{(1)}(z-a) = \Delta^{(1)}(a-z)$, the points x_i are connected by $\Delta^{\text{ret}}(a_i - x_i)$; a, a_i being some inner or outer points.

The $\delta/\delta(\frac{\delta}{\delta j'(y_i)})$ in the sin can only act on $\frac{\delta^m}{\delta j'(z_i)^m}$, such that by expanding sin we can make the replacement

$$\int dy_i\, 2\sin\left(\frac{1}{2}\frac{\delta}{\delta j(y_i)}\frac{\delta}{\delta\frac{\delta}{\delta j'(y_i)}}\right)\int dz_i\, g^m \frac{\delta^m}{\delta j'(z_i)^m}$$
$$\equiv 2\sum_{j\leqslant[\frac{m-1}{2}]} g^m \int dz_i\, \frac{1}{(2j+1)!}\left(\frac{1}{2}\right)^{2j+1}\frac{\delta^{2j+1}}{\delta j(z_i)^{2j+1}}\frac{\delta^{m-2j-1}}{\delta j'(z_i)^{m-2j-1}} \tag{13}$$

such that at the vertex z_i we have an odd power of $\frac{\delta}{\delta j(z_i)}$. As an incoming $\Delta^{\mathrm{ret}}(a-z_i)$ at vertex z_i can only be created by $\frac{\delta}{\delta j(z_i)}$ and vice versa, we find that the number of incoming Δ^{ret}-functions at each vertex must be odd.

One checks that there are no further restrictions to diagrams as the ones mentioned above, so we have found the diagrammatic rules for the retarded function $r(x; x_1, \dots, x_n)$:

1. $x, x_1, \dots, x_n$ are the endpoints of the diagram, inner points are called vertices.
2. $\Delta^{\mathrm{ret}}(x-y)$ is symbolized by $x \longrightarrow y$ $\Delta^{(1)}(x-y) = \Delta^{(1)}(y-x)$ by $x \text{———} y$.
3. x is connected by one line, $\Delta^{\mathrm{ret}}(x-a)$ or $\Delta^{(1)}(x-a)$. The points x_i are also connected by one line each, $\Delta^{\mathrm{ret}}(a_i - x_i)$.
4. The number of lines at each vertex is m for ϕ^m-theory, the contributing factor g, one integrates over the vertices.
5. The number of incoming functions $\Delta^{\mathrm{ret}}(a_i - z_i)$ at each vertex z_i is odd.

3. The noncommutative case

We implement noncommutativity by defining retarded functions via the generating functional (7), where the interaction now involves the star product, e.g., in noncommutative $\phi^3_\star$-theory $S_{\mathrm{int}} = \frac{g}{3!}\int dx\,(\phi\star\phi\star\phi)(x)$. This results in star multiplication at each vertex. In the Fourier representation of the retarded functions we thus encounter at every vertex a noncommutative phase factor $V(\pm p_1, \dots, \pm p_m)$ if $p_1, \dots, p_m$ are the momenta flowing $\left\{\begin{smallmatrix}\text{in}\\ \text{out}\end{smallmatrix}\right\}$ of the vertex. This phase factor is given by the m-point function at first order, e.g., in ϕ^3-theory it reads

$$V(p_1, p_2, p_3) = \frac{1}{6}\sum_{\pi\in S_3} e^{-i(p_{\pi(1)}, p_{\pi(2)}, p_{\pi(3)})}. \tag{14}$$

Here we made use of the abbreviation

$$(p_1, \dots, p_n) = \sum_{i<j} p_i \wedge p_j, \tag{15}$$

where the $\wedge$-product is defined as $p\wedge q = \frac{i}{2}p_\mu\theta^{\mu\nu}q_\nu$.

In space–time noncommutative theories this way of introducing retarded functions will not respect their support properties, i.e., in general we will also outside the region $x^0 \geqslant x_1^0, \dots, x_n^0$ have non-vanishing $r(x; x_1, \dots, x_n)$. This is due to the fact that for $\theta^{0i} \neq 0$ the star product involves time-derivatives, such that one smears over the time coordinate. It is therefore clear that one can no longer obtain the so-defined retarded functions from retarded products of the form (5), as they were originally introduced. However, we still consider the theory worth to be further studied, and compute S-matrix elements by using the reduction formula.

To obtain diagrammatic rules for the noncommutative case, the ones from the previous subsection only have to be supplemented by the rule

6. At every vertex x we perform star multiplication with respect to x.

3.1. Unitarity

To analyze unitarity, we follow closely the presentation in [18]. There the generalized unitarity condition

$$\mathcal{R}[0,j]=1 \tag{16}$$

is derived which implies unitarity for the S-matrix. The analysis of this condition in noncommutative theories will be the aim of this section. We consider the case of ϕ^m-theory and start with performing a Taylor expansion of the first exponential in (7):

$$\mathcal{R}[0,j]=1+\sum_{n=1}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int dy_i\,2\sin\left(\frac{1}{2}\frac{\delta}{\delta j(y_i)}\frac{\delta}{\delta\frac{\delta}{\delta j'(y_i)}}\right)\int dz_i\,g^m\frac{\delta^m}{\delta j'(z_i)^m}$$
$$\times\exp\left\{\int dy\,dz\left(\frac{1}{4}j'(y)\Delta^{(1)}(y-z)j'(z)-j'(y)\Delta^{\mathrm{ret}}(y-z)j(z)\right)\right\}\Bigg|_{j'=0}. \tag{17}$$

Each factor in the nth term ($n\geqslant 1$) of the sum contains at least one functional derivative $\delta/\delta j(z_i)$ such that we obtain $\prod_{i=1}^{n}\int dx_i\,j'(x_i)\Delta^{\mathrm{ret}}(x_i-z_i)$ in front of the exponential, which does not vanish for $j'=0$ only if every factor is differentiated with some $\delta/\delta j'(z_j)$. This means that at each vertex z_i we have an ending $\Delta^{\mathrm{ret}}(a-z_i)$, and the point a must be again out of the $\{z_i\}_{i=1}^{n}$, which implies that we have a closed cycle of Δ^{ret}-functions, i.e., an expression of the form

$$\underset{\uparrow}{\Delta}^{\mathrm{ret}}(z_1-z_2)\overset{z_2}{\star}\Delta^{\mathrm{ret}}(z_2-z_3)\overset{z_3}{\star}\cdots\overset{z_k}{\star}\Delta^{\mathrm{ret}}(z_k-z_1)\overset{z_1}{\star}. \tag{18}$$

The last statement can be seen as follows: choose z_{i_1}, which appears in a function $\Delta^{\mathrm{ret}}(a-z_{i_1}), a$ among the z_i's, say $a=z_{i_2}$. Either $z_{i_2}=z_{i_1}$ and we have found a closed cycle, or $z_{i_2}\neq z_{i_1}$ in which case we proceed by finding z_{i_3} such that $\Delta^{\mathrm{ret}}(z_{i_3}-z_{i_2})$ appears. In the case $z_{i_3}=z_{i_1}$ or $z_{i_3}=z_{i_2}$ we are finished, otherwise we go on in the same way. The limited number of points $\{z_i\}_{i=1}^{n}$ implies that the procedure will stop and yield a closed cycle of Δ^{ret}-functions.

This means that the only terms which spoil the unitarity condition (16) contain a closed cycle of Δ^{ret}-functions. Let us first consider the case $\theta^{0i}=0$, where the star product does not involve time derivatives. From the support property

$$\Delta^{\mathrm{ret}}(x)\neq 0\quad\text{only for } x^0>0 \tag{19}$$

we find as a condition that (18) does not vanish

$$z_1^0>z_2^0>\cdots>z_k^0>z_1^0, \tag{20}$$

which cannot be fulfilled, meaning that (18) is zero.

However, in the general case of space–time noncommutativity, one can no longer use this argumentation, as then taking star products contains a smearing over the time coordinates. In fact, it was argued in [15], that, e.g., $\Delta^{\mathrm{ret}}(x)\star\Delta^{\mathrm{ret}}(-x)\neq 0$. The diagrams involving expressions (18) thus are the ones which violate unitarity if time does not commute with space.

3.2. Composite operators: equations of motion and currents

To derive equations of motion on the quantized level, i.e., on the level of retarded functions, we define retarded functions $r^{\mathcal{O}}(x;x_1,\ldots,x_n)$ for a composite operator $\mathcal{O}$ at place x and single fields at $x_1,\ldots,x_n$ in the following way. We differentiate the generating functional by $\delta/\delta j'(x)$ once for every single field appearing in $\mathcal{O}$ and by

$\frac{\delta^n}{\delta j(x_1)\cdots\delta j(x_n)}$. For $\mathcal{O}$ in the form $\mathcal{O} = D_1\phi \star D_2\phi \star \cdots \star D_k\phi$ with D_i differential operators this means

$$r^{D_1\phi\star D_2\phi\star\cdots\star D_k\phi}(x; x_1, \ldots, x_n) \equiv D_1 \frac{\delta}{\delta j'(x)} \star D_2 \frac{\delta}{\delta j'(x)} \star \cdots \star D_k \frac{\delta}{\delta j'(x)} \frac{\delta^n}{\delta j(x_1)\cdots\delta j(x_n)} \mathcal{R}[j', j]\bigg|_{j'=j=0}, \tag{21}$$

e.g.,

$$r^{\phi\star(\Box+m^2)\phi}(x; x_1, \ldots, x_n) \equiv \frac{\delta}{\delta j'(x)} \star \left(\Box_x + m^2\right) \frac{\delta}{\delta j'(x)} \frac{\delta^n}{\delta j(x_1)\cdots\delta j(x_n)} \mathcal{R}[j', j]\bigg|_{j'=j=0}. \tag{22}$$

Diagrammatic rules for $r^{\mathcal{O}}(x; x_1, \ldots, x_n)$ with $\mathcal{O} = D_1\phi \star D_2\phi \star \cdots \star D_k\phi$ can be easily read off, the only change to the previous rules lies in how the point x is treated, we therefore replace rule 3 by

3′. x is connected by k lines; the ith line carries $D_i\Delta^{\mathrm{ret}}(x - a_i)$ or $D_i\Delta^{(1)}(x - a_i)$. The points x_i are connected by one line each, $\Delta^{\mathrm{ret}}(b_i - x_i)$. Star multiplication with respect to x is performed.

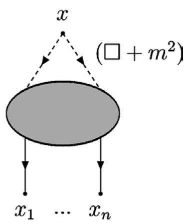
As an example of equations of motion and current conservation laws we now want to prove the bilinear equation of motion in $\phi^3_\star$-theory, which classically reads

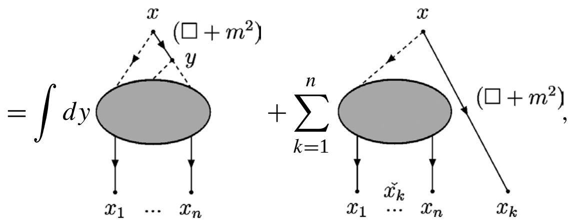
$$\phi \star \left(\Box + m^2\right)\phi = g\phi \star \phi \star \phi, \tag{23}$$

on the level of retarded functions, i.e., show that

$$r^{\phi\star(\Box+m^2)\phi}(x; x_1, \ldots, x_n) = r^{g\phi\star\phi\star\phi}(x; x_1, \ldots, x_n) + \text{c.t.}, \tag{24}$$

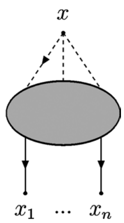
with c.t. meaning contact terms. We will evaluate both sides of the above equation diagrammatically:

$r^{\phi\star(\Box+m^2)\phi}(x; x_1, \ldots, x_n) =$ 



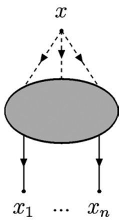
where the dashed arrow line can be or , and the dashed line stands for , or . We have used $(\Box + m^2)\Delta^{(1)}(x) = 0$ to skip diagrams that have a line $\Delta^{(1)}$ between x and y, respectively, x and x_k.

Applying $(\Box + m^2)\Delta^{\mathrm{ret}}(x) = \delta(x)$ we recognize the last diagram as contact terms, such that

$r^{\phi\star(\Box+m^2)\phi}(x; x_1, \ldots, x_n) =$  $+$ c.t. (25)

The right-hand side of Eq. (24) yields in terms of diagrams

$$r^{g\phi\star\phi\star\phi}(x;x_1,\ldots,x_n) = \tag{26}$$



To investigate under which conditions both sides are equal up to contact terms, we need to analyze under which conditions diagrams belonging to (25) with a dashed line being a Δ^{ret}-function that points to x are zero. At first, we prove the following

Lemma 1. *A diagram having at each vertex at least one incoming Δ^{ret}-function attached and the endpoints connected by outgoing Δ^{ret}-functions contains a closed cycle of Δ^{ret}-functions.*

Proof. Let $\{z_i\}_{i=1}^n$ be the set of vertices, at each z_i we have a function $\Delta^{\text{ret}}(a_i - z_i)$, and a_i must, as the outer points are connected by outgoing Δ^{ret}-functions, be itself out of $\{z_i\}_{i=1}^n$. We can now use the same argumentation as in the discussion of unitarity to obtain a closed cycle of Δ^{ret}-functions. □

If we consider the point x not as an endpoint but a vertex of the diagram, we find that diagrams belonging to (25) with a dashed line being a Δ^{ret}-function that points to x contain a closed cycle of Δ^{ret}-functions. From our discussion of closed cycles of Δ^{ret}-functions in the previous section we know that these vanish for $\theta^{0i} = 0$ but not necessarily otherwise. We have thus found that the classical bilinear equation of motion holds on the quantum level in the case of only spatial noncommutativity. However, it will be disturbed by diagrams containing closed cycles of Δ^{ret}-functions if time does not commute with space. This results generalizes to quantum current conservation laws, which are derived in a similar manner.

3.3. A modified theory

Let us first summarize our results so far. For space–time noncommutativity unitarity has turned out to be violated and the classical equations of motion and currents do not hold on the quantized level. In both cases these unpleasant outcomes are exactly due to diagrams which contain a closed cycle of Δ^{ret}-functions. Their vanishing for $\theta^{0i} = 0$ is the reason that in this case the approach via retarded functions yields a unitary theory and respects the classical equations.

The motivation to modify the theory is to obtain a theory which is unitary and preserves the classical equations of motion, therefore current conservation laws, on the tree-level and the finite part of the one-loop-level.

With the above results, it is obvious that we encounter these properties if we alter the theory by the requirement that we do not allow diagrams which exhibit a closed cycle of Δ^{ret}-functions. This modified theory can probably not be derived from a functional like (7), instead it is defined by the diagrammatic rules of Section 2.2 together with the rules of Section 3 and Section 3.2 if we impose the additional requirement

7. A diagram must not contain a closed cycle of Δ^{ret}-functions.

As diagrams which are excluded by the above rule vanish for $\theta^{0i} = 0$ the equivalence of the modified theory with the ordinary one derived from (7) in the case of only spatial noncommutativity is evident.

Let us briefly comment on Lorentz covariance: each diagram only involves expressions which are covariant under Lorentz transformations (if we also transform $\theta^{\mu\nu}$), thus are Lorentz-covariant. This property is therefore not disturbed by excluding a certain type of diagrams, meaning that the modified theory is still Lorentz covariant. We will thus expect it to respect remaining Lorentz symmetry.

4. Conclusions

We have extended retarded functions to noncommutative quantum field theories and analyzed the resulting perturbation theory. In space–time noncommutative theories we have found that unitarity is violated and the classical equations of motion and currents are not respected on the quantum level. Both unpleasant results can be ascribed to the same type of diagrams, which vanish in the case of only spatial noncommutativity. Modifying the theory by explicitly forbidding them yields a theory which has the desired properties of being unitary and respecting classical equations of motion and currents on the quantum level. This theory is defined by a set of diagrammatic rules, for vanishing θ^{0i} it coincides with the unmodified approach.

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